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Capability Evaluation of a Product Family for Processes of The Larger-the-Better Type

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Process capability indices (PCIs) have been widely used in manufacturing industry, but most of the studies associated with analysing the quality and efficiency of a process are limited to discussing one single quality specification. Practically, a product family is usually composed of several models, which result in different specifications. Generally, the quality characteristics of a product can be classified into three types; the nominal-the-best, the smaller-the-better and the larger-the-better types. This paper introduces one simple and applicable method to evaluate the process capability of a product family. The process is of the larger-the-better type and consists of several models with different specifications.

For practical applications, a simple step-by-step procedure is established to determine whether the total process capability of the product family meets the preset target. Finally, an example is given and a procedure for a hypothesis test is provided for easy application.

Keywords: Larger-the-better type; Process capability indices; Product family

1. Introduction

Process capability indices (PCIs) have proliferated in both use and variety during the last decade. They can provide the manufacturers with a means to monitor the quality levels of the procedures in process. Based on analysing the PCIs, a production department can trace and improve a poor process so that the quality level can be enhanced and the requirements of the customers can be satisfied. The process capability analysis can also serve as an important reference for making decisions for improving the global quality of all the products. Through the use of PCIs, the current status of a process can be monitored so that non-conforming products can be prevented and the quality of the products can be maintained above the required level. Furthermore, PCIs can serve as a communication medium for engineering designers and producers to reach rapid agreements so that an efficient system for quality improvement can be established.

PCIs are now widely used in many automated, semiconductor and IC assembly manufacturing industries to assure that the quality and efficiency of the processes are above the required level. Many statisticians and quality control engineers have studied the indices of processes so that the precision of assessing the quality and efficiency of a process can be enhanced. Many important results have thus been reported by Kane [1–8]. However, these studies have been limited to discussing the process capability of a single quality characteristic. Among the process capability indices, C_p, C_{pk}, C_{pm} and C_{pmk} are used for bilateral specifications. They are suitable for the processes of the nominal-the-best type. There are other indices, such as C_{pu} and C_{pl}. They are used for unilateral specification processes. The index C_{pu} is suitable for processes of the smaller-the-better type, whereas C_{pl} is suitable for processes of the larger-the-better type. These indices are defined as:

$$C_p = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma}$$

$$C_{pu} = \frac{\mathrm{USL} - \mu}{3\sigma}$$

$$C_{pl} = \frac{\mu - \mathrm{LSL}}{3\sigma}$$

$$C_{pk} = \min\{C_{pu}, C_{pl}\}$$

$$C_{pm} = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

$$C_{pmk} = \frac{\min\{\mathrm{USL} - \mu, \mu - \mathrm{LSL}\}}{3\sqrt{\sigma^2 + (\mu - T)^2}}$$

where μ is the process mean and σ is the process standard deviation, T is the target value, USL is the upper specification limit and LSL is the lower specification limit.

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In manufacturing industry, a product usually includes several models, which result in different specifications. Take hooks as an example. They are made to the customers' demands with many different specifications for carrying objects of various weights. However, there is still no effective tool for evaluating such a kind of process, which contains several models, i.e. the so-called "product family". Accreditation using Quality Assurance System QS9000 for quality and safety also requires an agent to conduct a global evaluation of the process capability over all the product family in the whole plant. When developing a new product or designing a new process, the indices are frequently used as references for important decision-making. Therefore developing a scheme to evaluate the process capability of a whole product family is very important for industry. The quality characteristics of many products, such as the hardness, or the tensile and compressive strengths, are of the larger-the-better type. These kinds of products have a lower specification limit. Therefore the index C_{pl} proposed by Kane [1] can be used to evaluate the process capability of each individual model. The process capability indices of all the individual models can be integrated for defining the process capability index of a product family of the larger-the-better type. This index has a mathematical relationship of a one-to-one mapping with the yields of the product. The yield of the whole product family can be back-calculated. The procedure would be as follows: (1) first, determine the lower specification limits of all the individual products; (2) collect the measured data for each quality characteristic; (3) use the measured data to estimate their capability indices C_{pl}. The yields can then be obtained from these indices as they have a one-to-one mapping relationship with the yields. All the yields of each individual product can be multiplied together to obtain the process yield of the whole product family. Once the overall index is determined, the manager can trace and analyse the process capability of an abnormal product by first analysing the process capability of the whole product family. In this way the best improvement can be made to the process in the minimum time. However, the estimates of the indices must be obtained from sampling because the parameters of the process are unknown. Therefore, it is not satisfactory to use only the estimates of these indices, to judge whether the quality and performance of the process meet the customers' demands, because of the errors induced by the sampling. In this paper, the relationship between the indices and the yields of a process will be further studied, and also statistical testing methods will be used to check whether the quality and performance of each process meet the customers' demands. Finally the quality and performance of k products will be evaluated with a table of checklists.

2. Capability Indices for a Product Family

Assume that a product of the larger-the-better type has k models. The process capability index for evaluating the ith model can be expressed by

$$C_{pli} = \frac{\mu_i - \mathrm{LSL}_i}{3\sigma_i},\ i = 1, 2, \ldots, k$$

where μ_i, σ_i and LSL_i are the process mean, the standard deviation and the lower specification limit of a product of the ith model, respectively. The specification, average, standard deviation and process capability index corresponding to each of the k models are listed in Table 1.

The process yield of a product of the larger-the-better is $P(X > \mathrm{LSL})$. Therefore, under the assumption of normal conditions, the relationship between the process yield p_i and the index C_{pli} of the ith model can be expressed by

$$p_i = \Phi(3C_{pli}),\ i = 1, 2, \ldots, k$$

where Φ is the standard cumulative normal distribution function. It is clear that the mathematic relationship between the index C_{pli} and the process yield p_i is one-to-one, as listed in Table 2. The yield is about 84.134% when the index value is 1/3, whereas the yield is up to about 99.865% when the index value is 1.0, i.e. the yield increases with the index value, and vice versa.

In practice, the yield of each model of a product is independent of each other. Let N_i denote the quantity of the ith product and therefore $N = \Sigma_{i=1}^{k} N_i$ denotes the total quantity of the whole product family. The total yield of the whole product family thus can be given by

$$p^T = \sum_{i=1}^{k} w_i \times p_i$$

where $w_i = N_i/N$ is the weight of each individual product.

The PCI for each product model can be measured by C_{pli}, and the PCI for the entire product family C_{pl}^T is the worst case among the k product models; that is, it is the minimum process capability of all product models. On the other hand, if the total process capability is C, then the individual process capability of product models will be at least greater than or equal to C. We now revise the relationship between C_{pl} and process yield: $p \geq \Phi(3C_{pl})$, proposed by Kotz [6], to reveal the connection between total process capability and the process yield for product family with k product models as follows: when

$$C_{pl}^T = \min\{C_{pl1}, C_{pl2}, \ldots, C_{plk}\} = C$$

then

$$C_{pli} \geq C,\ i = 1, 2, \ldots, k$$

The total process yield

$$p^T = \sum_{i=1}^{k} w_i p_i \geq \sum_{i=1}^{k} w_i\,[\Phi(3C)] = \Phi(3C)$$

Table 1. Specification, average, standard deviation, and process capability indices of a product family with k models.

Model	Specification	Mean	SD	Index value
1	LSL_1	μ_1	σ_1	C_{pl_1}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	LSL_i	μ_i	σ_i	C_{pl_i}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	LSL_k	μ_k	σ_k	C_{pl_k}

Table 2. Index values and the corresponding yields.

Index value	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000000	0.511966473	0.523922183	0.535856393	0.547758426	0.559617692	0.571423716	0.583166163	0.594834872	0.606419873
0.1	0.617911422	0.629300019	0.640576433	0.651731727	0.662757273	0.673644780	0.684386303	0.694974269	0.705401484	0.715661151
0.2	0.725746882	0.735652708	0.745373085	0.754902906	0.764237502	0.773372648	0.782304562	0.791029912	0.799545807	0.807849798
0.3	0.815939875	0.823814458	0.831472393	0.838912940	0.846135770	0.853140944	0.859928910	0.866500487	0.872856849	0.878999516
0.4	0.884930330	0.890651448	0.896165319	0.901474671	0.906582491	0.911492009	0.916206678	0.920730159	0.925066300	0.929219123
0.5	0.933192799	0.936991636	0.940620059	0.944082597	0.947383862	0.950528532	0.953521342	0.956367063	0.959070491	0.961636430
0.6	0.964069681	0.966375031	0.968557237	0.970621020	0.972571050	0.974411940	0.976148236	0.977784406	0.979324837	0.980773828
0.7	0.982135579	0.983414193	0.984613665	0.985737882	0.986790616	0.987775527	0.988696156	0.989555923	0.990358130	0.991105957
0.8	0.991802464	0.992450589	0.993053149	0.993612845	0.994132258	0.994613854	0.995059984	0.995472889	0.995854699	0.996207438
0.9	0.996533026	0.996833284	0.997109932	0.997364598	0.997598818	0.997814039	0.998011624	0.998192856	0.998358939	0.998511001
1.0	0.998650102	0.998777231	0.998893315	0.998999218	0.999095745	0.999183648	0.999263625	0.999336325	0.999402352	0.999462263
1.1	0.999516576	0.999565770	0.999610288	0.999650537	0.999686894	0.999719707	0.999749293	0.999775947	0.999799936	0.999821509
1.2	0.999840891	0.999858289	0.999873892	0.999887873	0.999900389	0.999911583	0.999921586	0.999930517	0.999938483	0.999945582
1.3	0.999951904	0.999957527	0.999962525	0.999966963	0.999970901	0.999974391	0.999977482	0.999980217	0.999982635	0.999984770
1.4	0.999986654	0.999988315	0.999989779	0.999991066	0.999992199	0.999993193	0.999994066	0.999994831	0.999995502	0.999996089
1.5	0.999996602	0.999997051	0.999997442	0.999997784	0.999998081	0.999998340	0.999998566	0.999998761	0.999998931	0.999999079
1.6	0.999999207	0.999999317	0.999999413	0.999999496	0.999999567	0.999999629	0.999999682	0.999999728	0.999999767	0.999999801
1.7	0.999999830	0.999999855	0.999999877	0.999999895	0.999999911	0.999999924	0.999999935	0.999999945	0.999999954	0.999999961
1.8	0.999999967	0.999999972	0.999999976	0.999999980	0.999999983	0.999999986	0.999999988	0.999999990	0.999999991	0.999999993
1.9	0.999999994	0.999999995	0.999999996	0.999999996	0.999999997	0.999999998	0.999999998	0.999999998	0.999999999	0.999999999

We conclude that $p^T \geq \Phi(3C)$. So, if the capability of the total product family, which equals the worst process capability among all product models, is obtained, then the total process yield is ensured. For example, if the capability of the total product family is 1.0, then it guarantees that the total process yield is at least greater than 0.99865.

3. Estimation of Capability Indices

With the assumption of a normal population, a set of random samples can be picked from the products of the ith model with a sample size of n. There are k models in the whole product family. Let μ_i and σ_i denote the mean and the standard deviation of the k models of the larger-the-better type, respectively. These values, as well as the estimators of the k models, are also listed in Table 3.

From the table, it is known that $\bar{X}_i = (\Sigma_{j=1}^{n} X_{ij})/n$ and $S_i = \{(n-1)^{-1} \sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2\}^{1/2}, i = 1, \ldots, k,$ are the natural estimators for the mean values and standard deviations. Evidently, the natural estimator of the total capability index C_{pl}^T can be expressed by

$$\hat{C}_{pl}^T = \min\{\hat{C}_{pl1}, \hat{C}_{pl2}, \ldots, \hat{C}_{plk}\}$$

where

$$\hat{C}_{pli} = (b_n) \times \frac{\bar{X}_i - L_i}{3S_i}, \; i = 1, 2, \ldots, k$$

The correction factor $b_n = [2/(n-1)]^{1/2}\, \Gamma[(n-1)/2]/\Gamma[(n-2)/2]$, and $\hat{C}_{pli}$ is the uniformly minimum variance unbiased estimator (UMVUE) of C_{pli} [9]. By using the theorem $f_{\hat{C}_{pl}^T}(y) = n\,[1 - F(y)]^{n-1} f(y)$ from Roussas [10], the probability density function of $\hat{C}_{pl}^T$ becomes:

$$f_{\hat{C}_{pl}^T}(y) = k\,[1 - F(y)]^{k-1} f_{\hat{C}_{pl_i}}(y)$$

where

$$F(y) = \int_0^y f_{\hat{C}_{pli}}(t)\,\mathrm{d}t$$

and

$$f_{\hat{C}_{pl_i}}(y) = \left(\frac{b_n^{-1} \times \sqrt{n} \times 2^{-(n/2)}}{3 \times \Gamma[(n-1)/2]}\right) \int_0^\infty t^{\left(\frac{n-2}{2}\right)} \exp\left\{-0.5\left[t + \left(\frac{\sqrt{nt}}{(n-1)b_n}\left(\frac{1}{3}\right) y - \delta\right)^2\right]\right\} \mathrm{d}t,$$

where

$$\delta = 3\sqrt{n}\, C_{pli}$$

Thus,

$$f_{\hat{C}_{pl}^T}(y) = k\left[1 - \int_0^y f_{\hat{C}_{pl}}(t)\,\mathrm{d}t\right]^{k-1} \times \left(\frac{b_n^{-1} \times \sqrt{n} \times 2^{-(n/2)}}{3 \times \Gamma[(n-1)/2]}\right) \int_0^\infty t^{\left(\frac{n-2}{2}\right)} \exp\left\{-0.5\left[t + \left(\frac{\sqrt{nt}}{(n-1)b_n}\left(\frac{1}{3}\right) y - \delta\right)^2\right]\right\} \mathrm{d}t$$

4. Testing of Indices

Cheng [11] pointed out that an estimation of the indices must be obtained from sampling because the parameters of the process are unknown; also, because of the errors produced by sampling, it is not satisfactory to use only the estimates of the indices for judging whether a process capability has met the customers' demands. Statistical hypothesis testing is one of the objective methods available to evaluate the capability of a process. This method can be used to examine whether the process capability of a specific model has met the customers' demands. On the basis mentioned above, the value of the PCI of a product family is guaranteed to be C if the PCIs of all individual models are controlled to be C.

Therefore we need to test whether the value of the PCI of each specific model is greater or equal to C. The hypothesis for testing can be stated as follows:

$$H_0: C_{pl}^T \geq C$$

$$H_1: C_{pl}^T < C$$

We reject the null hypothesis $C_{pl}^T \geq C$ (and accept the alternative $C_{pl}^T < C$) if the minimum PCI $\hat{C}_{pli}$ among samples from k product models is less than C_0, the critical point; otherwise, use reverse judgment. We intend to reject the alternative hypothesis to demonstrate that the total PCI for the product family is above the acceptable level. To determine whether the total product capability meets the preset target, the capable

Table 3. Model number, index, random samples, mean, standard deviation, and estimator of k models.

Model No.	Index	Random samples	Mean	SD	Estimator
1	C_{pl1}	$X_{11}, \ldots, X_{1n}$	$\bar{X}_1$	S_1	$\hat{C}_{pl1}$
⋮	⋮	⋮	⋮	⋮	⋮
i	C_{pli}	$X_{i1}, \ldots, X_{in}$	$\bar{X}_i$	S_i	$\hat{C}_{pli}$
⋮	⋮	⋮	⋮	⋮	⋮
k	C_{plk}	$X_{k1}, \ldots, X_{kn}$	$\bar{X}_k$	S_k	$\hat{C}_{plk}$

level C and the significant level α-risk are first decided. Then, we need to calculate each PCI $\hat{C}_{pli}$ from k sets of collected samples. The null hypothesis demonstrates that the total process capability is at least above the minimum tolerance level if once $\hat{C}_{pli}$ is greater than the critical value; otherwise, we reverse the conclusion. The significance level is α, i.e,

$$p(\hat{C}_{pl}^{T} < C_0 | C_{pl}^{T} \geq C) = \alpha$$
$$\Rightarrow p\ (\hat{C}_{pl}^{T} \geq C_0 | C_{pl}^{T} \geq C) = 1 - \alpha$$
$$\Rightarrow p(\min\{\hat{C}_{pl1}, \hat{C}_{pl2}, \ldots, \hat{C}_{plk}\} \geq C_0 | C_{pl1} \geq C, C_{pl2} \geq C, \ldots, C_{plk} \geq C) = 1 - \alpha$$
$$\Rightarrow P(3\sqrt{n}\hat{C}_{pli}/b_n \geq 3\sqrt{n}C_0/b_n | C_{pli} \geq c_0) = \sqrt[k]{1 - \alpha}$$
$$\Rightarrow P(t'(n - 1; \delta = 3\sqrt{n}c) \geq 3\sqrt{n}C_0/b_n) = \alpha'$$

where

$$\alpha' = \sqrt[k]{1 - \alpha}$$
and $t'(n - 1; \alpha = 3\sqrt{n}c) = 3\sqrt{n}\hat{C}_{pli}/b_n$.

This is a non-central t-distribution with degrees of freedom $n - 1$. The non-central parameter of this distribution is $\delta = 3\sqrt{n}C_{pli}$. Thus, we have

$$C_0 = \frac{t'_{\alpha'}\ (n - 1; \delta)}{3\sqrt{n}} \times b_n$$

where $t'_{\alpha'}(n - 1; \delta)$ is the upper α' percentile of $t'(n - 1; \delta)$.

Thus, the null hypothesis is rejected when $\hat{C}_{pl}^{T} < C_0$, which indicates that the process capability for the entire product family is less than the preset value. To demonstrate a high process capability, the value of $\hat{C}_{pli}$ needs to be large; that is, a higher individual PCI is required for each product model. Appendix Tables A–H display the critical value C_0 based on α level, sample size n and the capable process capability value C.

The complete testing procedure is summarized as follows in steps:

Step 1: Determine the value of C, the α-risk (normally set to 0.05) and the sample size n_i for each product model.

Step 2: Calculate the process capability index $\hat{C}_{pli}$ for each product model and the $\hat{C}_{pl}^{T} = \min\{\hat{C}_{pl1}, \hat{C}_{pl2}, \ldots, \hat{C}_{plk}\}$ for the entire product family.

Step 3: Calculate the critical value C_0.

Step 4: Make a decision about whether the total process capability for the entire product family is sufficient by comparing $\hat{C}_{pl}^{T}$ with the critical value C_0. If $\hat{C}_{pl}^{T}$ is greater than C_0, then the conclusion is that the process capability for the entire product family meets the preset target; otherwise, it fails the requirement.

5. An application

To illustrate how the procedure may be applied to real cases where data would actually be collected from the process, a case study on a crane hook manufacturing process is presented. In this case, a product family with eight models of crane hooks, 8006, 8007, 8010, 8013, 8016, 8018, 8022, 8026, is produced with different specifications. The specifications corresponding to each of the eight models are listed in Table 4. The critical measurement is how much strength is needed to meet the customer's requirement. The quality characteristic of this product family is clearly the larger-the-better type. The complete testing procedure is summarised in the following steps:

Step 1: The sufficient process capability value is determined as $C = 1.33$, the significance level is 0.05 and the sample size is $n = 50$ for all product models.

Step 2: Calculate the value of the estimator $\hat{C}_{pli}$ from the sample and insert the results in Table 4. After all the estimators $\hat{C}_{pli}$ for the entire product family having been calculated, the estimator $\hat{C}_{pl}^{T}$ can be obtained through $\hat{C}_{pl}^{T} = \min\{1.201, 1.220, 1.090, 1.160, 1.254, 1.018, 1.305, 1.180\} = 1.018$.

Step 3: Check the appropriate table (Table C) and find the corresponding critical value $C_0 = 1.025$ based on $\alpha = 0.05$, $C = 1.33$, $n = 50$ and $k = 8$.

Step 4: By comparing $C_{pl}^{T} = 1.018$ with the critical value $C_0 = 1.025$, the total process capability for the entire product family does not meet the preset target. Therefore prompt and proper process modifications and calibrations on model 8018 must be undertaken.

6. Conclusion

PCI are convenient and efficient tools for evaluating one single quality characteristic. Several PCIs have been widely used to measure whether process quality meets the preset target. However, those existing PCIs cannot be applied to a product family and there is no suitable methodology to evaluate a product family with different models. In this paper we remove the limitations of process capability indices for dealing with a product family with several product models. Those products are all the same in function and design; the only differences

Table 4. The specifications of a product family with eight models.

No.	Model	LSL (lb)	$\bar{X}_i$	S_i	$\hat{C}_{pli}$
1	8006	8400	8850	123	1.201
2	8007	14000	14520	140	1.220
3	8010	28400	28815	125	1.090
4	8013	48000	48470	133	1.160
5	8016	72400	72820	110	1.254
6	8018	113200	113628	138	1.018
7	8022	136800	137245	112	1.305
8	8026	190800	191285	135	1.180

$\hat{C}_{pl}^{T} = \min\{1.201, 1.220, 1.090, 1.160, 1.254, 1.018, 1.305, 1.180\} = 1.018$

are the sizes. To evaluate yield, a PCI is proposed in this paper for evaluating a product family of the larger-the-better type. The proposed index C_{pl}^{T} can evaluate the capability and also evaluate the yield of a process because mathematically it has a one-to-one relationship with the yield. It is indeed a good index. The testing procedures for using the results of this paper to evaluate the capability of a process of the larger-the-better type are also included. With the aid of a check table, the quality and performance of a process with k models can be assessed. Managers in industry can apply this method to analyse the process capability of a product family. The answer as to whether the process capability of the whole product family has met the requirements can thus be obtained rapidly, and then the process capability of each abnormal product can be analysed. In this way, the best improvements can be achieved with the least amount of time and effort.

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Table A. The critical value C_0 based on $\alpha = 0.05$, $C = 1.00$.

n	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$
10	0.634	0.592	0.570	0.556	0.546	0.538	0.531	0.526	0.521
15	0.699	0.660	0.640	0.627	0.618	0.610	0.604	0.599	0.595
20	0.737	0.701	0.683	0.671	0.662	0.655	0.649	0.645	0.640
25	0.764	0.730	0.713	0.702	0.693	0.687	0.681	0.677	0.673
30	0.783	0.752	0.735	0.725	0.717	0.710	0.705	0.701	0.697
35	0.798	0.768	0.753	0.743	0.735	0.729	0.724	0.720	0.717
40	0.811	0.782	0.767	0.758	0.750	0.745	0.740	0.736	0.733
45	0.821	0.794	0.779	0.770	0.763	0.758	0.753	0.749	0.746
50	0.830	0.803	0.790	0.781	0.774	0.769	0.764	0.761	0.757
55	0.837	0.812	0.799	0.790	0.783	0.778	0.774	0.770	0.767
60	0.844	0.819	0.807	0.798	0.792	0.787	0.783	0.779	0.776
65	0.850	0.826	0.813	0.805	0.799	0.794	0.790	0.787	0.784
70	0.855	0.832	0.820	0.812	0.806	0.801	0.797	0.794	0.791
75	0.860	0.837	0.825	0.818	0.812	0.807	0.803	0.800	0.797
80	0.864	0.842	0.830	0.823	0.817	0.813	0.809	0.806	0.803
85	0.868	0.846	0.835	0.828	0.822	0.818	0.814	0.811	0.808
90	0.871	0.850	0.839	0.832	0.827	0.822	0.819	0.816	0.813
95	0.875	0.854	0.843	0.836	0.831	0.827	0.823	0.820	0.818
100	0.878	0.858	0.847	0.840	0.835	0.831	0.827	0.824	0.822

Table B. The critical value C_0 based on $\alpha = 0.10$, $C = 1.00$.

n	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$
10	0.690	0.636	0.610	0.593	0.581	0.571	0.564	0.557	0.552
15	0.748	0.701	0.677	0.661	0.650	0.641	0.634	0.628	0.623
20	0.783	0.739	0.717	0.703	0.692	0.684	0.678	0.672	0.667
25	0.805	0.765	0.745	0.731	0.722	0.714	0.708	0.703	0.698
30	0.822	0.784	0.765	0.753	0.744	0.736	0.731	0.726	0.721
35	0.835	0.800	0.781	0.770	0.761	0.754	0.748	0.744	0.740
40	0.846	0.812	0.795	0.783	0.775	0.768	0.763	0.759	0.755
45	0.855	0.822	0.806	0.795	0.787	0.780	0.775	0.771	0.767
50	0.862	0.831	0.815	0.804	0.797	0.791	0.786	0.782	0.778
55	0.868	0.838	0.823	0.813	0.805	0.800	0.795	0.791	0.787
60	0.874	0.845	0.830	0.820	0.813	0.807	0.803	0.799	0.795
65	0.879	0.851	0.836	0.827	0.820	0.814	0.810	0.806	0.803
70	0.883	0.856	0.842	0.833	0.826	0.820	0.816	0.812	0.809
75	0.887	0.861	0.847	0.838	0.831	0.826	0.822	0.818	0.815
80	0.891	0.865	0.851	0.843	0.836	0.831	0.827	0.824	0.820
85	0.894	0.869	0.856	0.847	0.841	0.836	0.832	0.828	0.825
90	0.897	0.872	0.860	0.851	0.845	0.840	0.836	0.833	0.830
95	0.899	0.875	0.863	0.855	0.849	0.844	0.840	0.837	0.834
100	0.902	0.878	0.866	0.858	0.852	0.848	0.844	0.841	0.838

Table C. The critical value C_0 based on $\alpha = 0.05$, $C = 1.33$.

n	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$
10	0.862	0.808	0.781	0.764	0.751	0.741	0.733	0.726	0.720
15	0.945	0.896	0.872	0.855	0.843	0.834	0.826	0.820	0.815
20	0.995	0.949	0.926	0.911	0.900	0.891	0.884	0.878	0.873
25	1.028	0.986	0.964	0.950	0.940	0.931	0.925	0.919	0.914
30	1.053	1.013	0.993	0.979	0.969	0.962	0.955	0.950	0.945
35	1.073	1.035	1.015	1.003	0.993	0.986	0.979	0.974	0.970
40	1.089	1.052	1.034	1.021	1.012	1.005	0.999	0.994	0.990
45	1.102	1.067	1.049	1.037	1.028	1.022	1.016	1.011	1.007
50	1.113	1.080	1.062	1.051	1.042	1.036	1.030	1.025	1.021
55	1.122	1.090	1.074	1.063	1.054	1.048	1.042	1.038	1.034
60	1.131	1.100	1.084	1.073	1.065	1.059	1.053	1.049	1.045
65	1.138	1.108	1.092	1.082	1.074	1.068	1.063	1.059	1.055
70	1.145	1.116	1.100	1.090	1.083	1.077	1.072	1.068	1.064
75	1.151	1.122	1.108	1.098	1.090	1.084	1.080	1.076	1.072
80	1.156	1.129	1.114	1.104	1.097	1.091	1.087	1.083	1.079
85	1.161	1.134	1.120	1.111	1.103	1.098	1.093	1.089	1.086
90	1.166	1.139	1.125	1.116	1.109	1.104	1.099	1.096	1.092
95	1.170	1.144	1.131	1.121	1.115	1.109	1.105	1.101	1.098
100	1.174	1.149	1.135	1.126	1.120	1.114	1.110	1.107	1.103

Table D. The critical value C_0 based on $\alpha = 0.10$, $C = 1.33$.

n	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$
10	0.932	0.864	0.831	0.810	0.795	0.783	0.773	0.765	0.759
15	1.008	0.947	0.917	0.898	0.884	0.873	0.864	0.857	0.850
20	1.052	0.997	0.969	0.951	0.938	0.928	0.920	0.913	0.907
25	1.081	1.030	1.004	0.988	0.975	0.966	0.958	0.951	0.946
30	1.103	1.055	1.031	1.015	1.003	0.994	0.987	0.981	0.975
35	1.120	1.074	1.051	1.036	1.025	1.017	1.010	1.004	0.999
40	1.133	1.090	1.068	1.054	1.043	1.035	1.028	1.023	1.018
45	1.144	1.103	1.082	1.068	1.058	1.050	1.044	1.038	1.034
50	1.154	1.114	1.094	1.081	1.071	1.063	1.057	1.052	1.047
55	1.162	1.124	1.104	1.091	1.082	1.075	1.069	1.064	1.059
60	1.169	1.132	1.113	1.101	1.092	1.085	1.079	1.074	1.070
65	1.175	1.140	1.121	1.109	1.100	1.093	1.088	1.083	1.079
70	1.181	1.146	1.128	1.117	1.108	1.101	1.096	1.091	1.087
75	1.186	1.152	1.135	1.123	1.115	1.108	1.103	1.099	1.095
80	1.190	1.158	1.141	1.130	1.121	1.115	1.110	1.105	1.101
85	1.194	1.162	1.146	1.135	1.127	1.121	1.116	1.111	1.108
90	1.198	1.167	1.151	1.140	1.132	1.126	1.121	1.117	1.113
95	1.202	1.171	1.155	1.145	1.137	1.131	1.126	1.122	1.119
100	1.205	1.175	1.160	1.149	1.142	1.136	1.131	1.127	1.124

Table E. The critical value C_0 based on $\alpha = 0.05$, $C = 1.50$.

n	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$
10	0.978	0.918	0.889	0.869	0.855	0.844	0.835	0.828	0.821
15	1.071	1.017	0.989	0.971	0.958	0.948	0.939	0.932	0.926
20	1.126	1.076	1.050	1.034	1.021	1.012	1.004	0.997	0.991
25	1.164	1.117	1.093	1.077	1.065	1.056	1.049	1.042	1.037
30	1.192	1.147	1.125	1.110	1.099	1.090	1.083	1.077	1.071
35	1.213	1.171	1.150	1.135	1.125	1.116	1.110	1.104	1.099
40	1.231	1.191	1.170	1.156	1.146	1.138	1.132	1.126	1.121
45	1.246	1.207	1.187	1.174	1.164	1.157	1.150	1.145	1.140
50	1.258	1.221	1.202	1.189	1.180	1.172	1.166	1.161	1.156
55	1.269	1.233	1.214	1.202	1.193	1.186	1.180	1.175	1.171
60	1.278	1.244	1.226	1.214	1.205	1.198	1.192	1.187	1.183
65	1.286	1.253	1.235	1.224	1.215	1.208	1.203	1.198	1.194
70	1.294	1.261	1.244	1.233	1.225	1.218	1.212	1.208	1.204
75	1.301	1.269	1.252	1.241	1.233	1.227	1.221	1.217	1.213
80	1.307	1.276	1.259	1.249	1.241	1.234	1.229	1.225	1.221
85	1.312	1.282	1.266	1.256	1.248	1.242	1.237	1.232	1.228
90	1.317	1.288	1.272	1.262	1.254	1.248	1.243	1.239	1.235
95	1.322	1.293	1.278	1.268	1.260	1.254	1.249	1.245	1.242
100	1.326	1.298	1.283	1.273	1.266	1.260	1.255	1.251	1.248

Table F. The critical value C_0 based on $\alpha = 0.10$, $C = 1.50$.

n	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$
10	1.056	0.981	0.944	0.921	0.904	0.890	0.880	0.871	0.863
15	1.141	1.074	1.040	1.019	1.003	0.991	0.981	0.973	0.966
20	1.190	1.128	1.098	1.078	1.063	1.052	1.043	1.035	1.028
25	1.223	1.166	1.137	1.119	1.105	1.094	1.086	1.078	1.072
30	1.247	1.194	1.167	1.149	1.136	1.126	1.118	1.111	1.105
35	1.266	1.215	1.190	1.173	1.161	1.151	1.143	1.137	1.131
40	1.281	1.233	1.208	1.192	1.181	1.171	1.164	1.158	1.152
45	1.293	1.247	1.224	1.209	1.197	1.189	1.181	1.175	1.170
50	1.304	1.260	1.237	1.222	1.212	1.203	1.196	1.190	1.185
55	1.313	1.270	1.249	1.234	1.224	1.216	1.209	1.203	1.198
60	1.321	1.280	1.259	1.245	1.235	1.227	1.220	1.215	1.210
65	1.328	1.288	1.267	1.254	1.244	1.237	1.230	1.225	1.220
70	1.334	1.295	1.275	1.262	1.253	1.245	1.239	1.234	1.229
75	1.339	1.302	1.283	1.270	1.261	1.253	1.247	1.242	1.238
80	1.344	1.308	1.289	1.277	1.268	1.261	1.255	1.250	1.245
85	1.349	1.313	1.295	1.283	1.274	1.267	1.261	1.257	1.252
90	1.353	1.318	1.301	1.289	1.280	1.273	1.268	1.263	1.259
95	1.357	1.323	1.306	1.294	1.285	1.279	1.273	1.269	1.265
100	1.361	1.327	1.310	1.299	1.291	1.284	1.279	1.274	1.270

Table G. The critical value C_0 based on $\alpha = 0.05$, $C = 2.00$.

n	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$
10	1.317	1.239	1.201	1.176	1.157	1.143	1.132	1.122	1.113
15	1.439	1.369	1.333	1.310	1.292	1.279	1.268	1.259	1.251
20	1.511	1.446	1.413	1.391	1.375	1.362	1.352	1.344	1.336
25	1.561	1.500	1.468	1.448	1.433	1.421	1.411	1.403	1.396
30	1.597	1.540	1.510	1.490	1.476	1.465	1.455	1.448	1.441
35	1.626	1.571	1.543	1.524	1.510	1.499	1.491	1.483	1.477
40	1.649	1.596	1.569	1.552	1.538	1.528	1.519	1.512	1.506
45	1.668	1.618	1.592	1.574	1.562	1.552	1.544	1.537	1.531
50	1.684	1.636	1.611	1.594	1.582	1.572	1.564	1.558	1.552
55	1.698	1.651	1.627	1.611	1.599	1.590	1.582	1.576	1.570
60	1.710	1.665	1.642	1.626	1.615	1.606	1.598	1.592	1.586
65	1.721	1.677	1.655	1.639	1.628	1.619	1.612	1.606	1.601
70	1.731	1.688	1.666	1.651	1.640	1.632	1.625	1.619	1.614
75	1.739	1.698	1.676	1.662	1.651	1.643	1.636	1.630	1.625
80	1.747	1.707	1.686	1.672	1.662	1.653	1.647	1.641	1.636
85	1.755	1.715	1.695	1.681	1.671	1.663	1.656	1.650	1.646
90	1.761	1.723	1.703	1.689	1.679	1.671	1.665	1.659	1.655
95	1.767	1.730	1.710	1.697	1.687	1.679	1.673	1.668	1.663
100	1.773	1.736	1.717	1.704	1.694	1.687	1.680	1.675	1.671

Table H. The critical value C_0 based on $\alpha = 0.10$, $C = 2.00$.

n	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$
10	1.418	1.320	1.272	1.242	1.220	1.203	1.189	1.178	1.168
15	1.530	1.442	1.399	1.371	1.351	1.335	1.322	1.312	1.302
20	1.594	1.514	1.474	1.448	1.430	1.415	1.403	1.393	1.384
25	1.638	1.563	1.526	1.502	1.484	1.470	1.459	1.450	1.442
30	1.669	1.600	1.564	1.542	1.525	1.512	1.501	1.492	1.484
35	1.694	1.628	1.595	1.573	1.557	1.544	1.534	1.526	1.518
40	1.713	1.651	1.619	1.598	1.583	1.571	1.561	1.553	1.546
45	1.730	1.670	1.639	1.619	1.605	1.593	1.584	1.576	1.569
50	1.743	1.686	1.657	1.637	1.623	1.612	1.603	1.596	1.589
55	1.755	1.700	1.672	1.653	1.639	1.629	1.620	1.613	1.606
60	1.765	1.712	1.685	1.667	1.654	1.643	1.635	1.628	1.621
65	1.775	1.723	1.696	1.679	1.666	1.656	1.648	1.641	1.635
70	1.783	1.732	1.707	1.690	1.677	1.667	1.659	1.653	1.647
75	1.790	1.741	1.716	1.700	1.687	1.678	1.670	1.663	1.658
80	1.797	1.749	1.725	1.708	1.697	1.687	1.680	1.673	1.668
85	1.803	1.756	1.732	1.717	1.705	1.696	1.688	1.682	1.677
90	1.808	1.763	1.739	1.724	1.713	1.704	1.697	1.690	1.685
95	1.813	1.769	1.746	1.731	1.720	1.711	1.704	1.698	1.693
100	1.818	1.774	1.752	1.737	1.726	1.718	1.711	1.705	1.700